Unravelling the origins of anomalous diffusion: From molecules to migrating storks

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Anomalous diffusion or, more generally, anomalous transport, with nonlinear dependence of the mean-squared displacement on the measurement time, is ubiquitous in nature. It has been observed in processes ranging from microscopic movement of molecules to macroscopic, large-scale paths of migrating birds. Using data from multiple empirical systems, spanning 12 orders of magnitude in length and 8 orders of magnitude in time, we employ a method to detect the individual underlying origins of anomalous diffusion and transport in the data. This method decomposes anomalous transport into three primary effects: long-range correlations ("Joseph effect"), fat-tailed probability density of increments ("Noah effect"), and nonstationarity ("Moses effect"). We show that such a decomposition of real-life data allows us to infer nontrivial behavioral predictions and to resolve open questions in the fields of single-particle tracking in living cells and movement ecology.

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I. INTRODUCTION

Normal diffusion or transport processes obey the Gaussian central limit theorem (CLT) and are ergodic, i.e., mean values of various observables in the system do not depend on the averaging method. The CLT states that if a random time series $x(t)$ is the sum of random variables which are (i) identically distributed (with a stationary distribution), (ii) have a finite variance, and (iii) are independent, then the probability density function (PDF) $P(x,t)$ of x at time t has a Gaussian shape (see Sec. III). The mean-squared displacement (MSD) then satisfies $\langle x^2(t)\rangle \propto t$ at long times, where $\langle\cdot\rangle$ denotes ensemble averaging (EA). Yet, advances in high-fidelity methods for single-particle tracking [1,2] and detailed data of animal paths [3,4] show that many natural processes are in fact *anomalous*, as they violate (some of) the CLT's conditions [5]. Condition (i) can be violated, e.g., when the measured trajectories are confined for increasingly long periods in certain spatial regions, hindering their expansion. Condition (ii) can be violated, e.g., in financial time series, where large fluctuations are highly probable. Condition (iii) can be violated, e.g., for biased or correlated motion. Such violations typically yield

$$\langle x^2(t)\rangle \propto t^{2H}, \tag{1}$$

with the Hurst exponent being $H \neq 1/2$.

Given an empirical time series displaying anomalous transport, the ability to distinguish between the various violations of the CLT is crucial, e.g., to determine the system's expansion rate [6,7], rare event statistics [8,9], and method of averaging [10–12], as well as to infer features in the diffusion medium [13–16] and elucidate the underlying microscopic processes. However, this characterization remains a major challenge in various fields, including single-particle tracking and movement ecology [17–19], and much effort is made to develop techniques to tackle it; see, e.g., Refs. [20–23]. Recently, machine-learning methods for analyzing anomalous transport data have been widely studied, see, e.g., Refs. [24,25], and for many applications they were shown to outperform estimators based on classical statistics [26]. Yet the "black box" nature of these data-driven algorithms may hinder the ability to account for the underlying reasons of the observed phenomena [26].

Here, based on positional (tracking) data, we employ a specialized three-effect decomposition method [27,28] to disentangle the effects leading to anomalous transport, without making prior assumptions on the underlying model governing the dynamics. By analyzing three independent properties of the time series presented below, we determine whether the

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